Physics 151 Class Exercise: Buoyancy - KEY

1. A cube of iron 0.300 m on a side is suspended (in equilibrium) from a large spring scale (which reads in Newtons) held over a giant tank of water. The center of the cube is 30.00 m below the surface of the water.

(a) What is the volume of the cube? $V = l^3 = (0.300m)^3 = 0.0270\,m^3$

(b) What is the mass of the cube? $M = \rho_{Fe}V = \left(7860\frac{kg}{m^3}\right)(0.0270m^3) = 212kg$

(c) What is the weight of the water displaced by the cube?

$$W = \rho_{Fe}Vg = \left(1000\frac{kg}{m^3}\right)(0.0270m^3)\left(9.81\frac{m}{s^2}\right) = 265\,N$$

(d) What is the magnitude of the force of buoyancy on the cube? $F_B = 265N$

(e) What is the reading on the spring scale ? $T = mg - F_B = (212kg)\left(9.81\frac{m}{s^2}\right) - 265N = 1810N$

(f) What is the pressure on the top surface of the cube?

$$P = P_a + \rho gh = (1.013\times10^5\,Pa) + \left(1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)(29.85m) = 3.941\times10^5\,Pa$$

(g) What is the pressure on the bottom surface of the cube?

$$P = P_a + \rho gh = (1.013\times10^5\,Pa) + \left(1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)(30.15m) = 3.971\times10^5\,Pa$$

(h) What is the force on the top surface of the cube?

$$F = PA = (3.941\times10^5\,Pa)(0.3m)^2 = 35470N$$

(i) What is the force on the bottom surface of the cube?

$$F = PA = (3.971\times10^5\,Pa)(0.3m)^2 = 35740N$$

(j) What is the difference between the force on the bottom and the force on the top?

$$\Delta F = F_1 - F_2 = 35740N - 35470N = 270N$$

2. A piece of lead has the shape of a hockey puck, with a diameter of 7.5 cm and a height of 2.5 cm. If the puck is placed in a mercury bath it floats. How deep below the surface of the mercury is the bottom of the lead puck?

Note that the cross-sectional area cancels out. Another example of the advantages of using variables.

$$\begin{aligned}
F_\text{b} &= W \\
\rho_\text{Hg} V_\text{submerged}\, g &= mg \\
\rho_\text{Hg} V_\text{submerged} &= \rho_\text{lead} V \\
\rho_\text{Hg}\left(\frac{\pi D^2 h_\text{submerged}}{4}\right) &= \rho_\text{lead}\left(\frac{\pi D^2 h}{4}\right) \\
h_\text{submerged} &= \frac{\rho_\text{lead}}{\rho_\text{Hg}} h \\
&= \frac{11.3\times10^3\ \frac{\text{kg}}{\text{m}^3}}{13.6\times10^3\ \frac{\text{kg}}{\text{m}^3}}(0.025\,\text{m}) \\
&= \boxed{2.1\,\text{cm}}
\end{aligned}$$